Archimedes's Impact on the Discovery Process in Ancient Greece

AHS Capstone Disciplinary Deliverable

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Introduction

"The transition from *praxis* to *theoria*—from mathematics as *tekhne* (techniques for dealing with practical activities) to mathematics as *episteme* and *gnosis* (a form of pure knowledge)—occurred only once in human history, namely, among the classical Greeks. No earlier mathematical tradition gives evidence of such a theoretical dimension, and where one encounters mathematical theory among later traditions, it is in the context of some manner of borrowing from the ancient Greek precedent"[1]

The goal of this paper is to examine the above "Greek precedent," their process of developing mathematics as well as other scientific theory. We will first examine the rise of rational thought and beginnings of deduction in the Archaic Era, using Thales as a representative of the time period. Then we will see how Aristotle added the syllogism and emphasis on observation during the Classical Era in order to enhance and codify deductive reasoning. Finally, Archimedes enters the picture and introduces an inductive method of discovery that crosses the disciplines of mathematics and mechanics. Throughout the paper are examples of how the Greeks applied their mathematical and scientific knowledge in order to demonstrate what new developments in reasoning added to the times.

Thales

Thales was a citizen of Miletus who lived in the Archaic Age of Greece. He was eventually considered one of the seven sages of Greece and was even regarded as the first philosopher and the father of science. He was the first Greek we know of who introduced the basis of rational thought, that natural phenomena were caused by underlying, general principles, not gods. As a mathematician, Thales discovered many fundamental concepts

[1] [Brunschwig and Lloyd] page 386

in geometry. He demonstrated such principles as the congruency of opposite angles, the congruency of base angles of an isosceles triangle, and that the diameter of a circle divides it into two equal sections. These findings seem rather obvious, but that Thales explicitly stated these conclusions makes it clear that deductive reasoning was developing. In this way he established foundational principles from which less obvious, more abstract results could be derived. But Thales also used his knowledge in practical ways. By using similar triangles when his shadow was equal to his length, he was able to calculate the height of the pyramids. He probably knew the path of the sun with respect to the earth because he set the year at 365 days. He also fixed the dates of the solstices and predicted eclipses. In order to show that philosophers were capable of making money, he bought all the olive presses in his city-state after predicting that the harvest for that year would be particularly fruitful. In this way he showed that application of theory could be useful concerning business as well. Thales embodied the transition to rational thought, the foundation of deductive reasoning. His appearance set the stage for theoreticians who would follow.

Aristotle

After the Archaic Era came the Classical Era, for which Aristotle was the indisputable icon of deductive reasoning. At the age of seventeen, Aristotle traveled to Athens to study at Plato’s Academy, where he remained for twenty years. During his stay, Aristotle introduced the most compelling form of reasoning up to that point, the syllogism. A syllogism is a set of three propositions in which one (the conclusion) is deduced entirely from the other two, which are assumed to be true. Thus the validity of the conclusion is based exclusively on the truth of the stated premises. While syllogisms are not necessarily tools of discovery, they do reveal what results from a set of principles assumed to be true. This is why Aristotle asserted definitions meticulously; in fact, he wrote at length about definitions in his treatise *Posterior Analytics*. A definition, in a way, can be an axiom—it involves an idea and affiliated beliefs taken as true. Aristotle also believed that there was "one science corresponding to one genus," or, category[2]. In other words, he believed that the results of one field of study could not be applied to another except through analogy, which may have limited interdisciplinary study.

Aristotle certainly added to deductive reasoning when he codified his approach, but his dependence on observation without experiment qualifies much of what he hypothesized. For instance, he says in *The History of Animals* that males have more teeth than females, which is untrue and perhaps based on a limited number of observations.[3]

Hellenistic Era

One year after Aristotle died (323 BC) came the next era in Greece: the Hellenistic Era. Hellenistic scientists gave birth to the experiment's role in development of theory. While Classical scientists were primarily concerned with observation and the deductive method, Hellenistic scientists oriented themselves towards the scientific method that we understand today.

In order to illustrate the difference, we can take an example of Archimedes refuting Aristotle.[4] Aristotle, through observation and philosophical deduction, had argued that the there could be no ratio of force to distance traveled by an object. Aristotle provided the example of a ship that took a multitude of men to move. He explained that if the hypothesis were true, it would imply that a single man could move the ship, but very

[2] [Brunshwig and Lloyd] page 570
[3] http://en.wikipedia.org/wiki/Aristotle, accessed April 29, 2006
[4] [Russo] pages 25-26

slowly. Since he had never observed this, he deemed it impossible.[5] Archimedes, on the other hand, by creating a system of simple machines that gave enough mechanical advantage, debunked Aristotle by moving a ship single-handedly.

Concerning mathematics, Euclid ushered in a new way of structuring the discipline.[6] He established a mere five assumptions (postulates), which resulted from the tools available at the time: the ruler and the compass. Since lines and circles could be drawn with these instruments, the postulates gave the base assumptions about these figures in an 'ideal' world. In his works on geometry, Euclid built up his theory solely from definitions and the original five postulates. In this way, he redefined the *apodeixis* ("proof") as the path one takes from these postulates to the theorem he was trying to prove—this was one of the first times that assumptions were stated so clearly.

Enter Archimedes

Into this Hellenistic world was born Archimedes. He had the tools of deduction, observation, and experiment at hand because of the work of his predecessors; but he would soon add to this array. Archimedes crafted many new inventions that granted him fame even though it *appears* (according to Plutarch) that his own motivations were primarily for the development of pure theory.[7] Although it still remains unclear whether Archimedes considered himself more of a theoretician or practitioner (or whether he even made this distinction), Plutarch cast Archimedes as an intransigent theoretician who "despised application."

Archimedes was in contact with the head of the Library of Alexandria, Eratosthenes of Cyrene, with whom he corresponded about mathematical theory, as we

[5] Aristotle, *Physica*, VII, v, 250a
[6] [Russo] pages 39-40
[7] Plutarch, *Vita Marcelli* XVII, 4 (307)

shall see below in an excerpt from his treatise on *Method*. Archimedes also wrote many treatises on scientific theory and even invented the field of hydrostatics. In his treatise *On Floating Bodies*, Archimedes laid out the fundamental principles of hydrostatics. While the book contains nothing explicit about the application of the theory, it was written such that an application to shipbuilding, for example, would be straightforward.

Archimedes was even the first to establish $22/7$ as an upper bound of the mathematical constant π. In order to do this, he employed the method of exhaustion, through which he ‘trapped’ π by both inscribing and circumscribing circles with polygons of higher and higher numbers of sides. Archimedes applied this method to find other geometrical results, particularly with respect to parabolas and ratios of sections therein.

In his mathematical treatise *Sand Reckoner,* Archimedes created a number system capable of counting all of the grains of sand in the universe. Before this work, the Greeks had not yet developed a way to count to such a large number. Archimedes’s new method of counting resembled our modern exponential notation, a major leap for mathematics. He also clarified the concept of the infinite, showing that even such a large calculation as the number of grains of sand in the world is finite.

Concerning mechanics, Archimedes recognized the importance of the center of gravity throughout his works, proving how it could be calculated for both two-dimensional and three-dimensional geometrical figures. Interestingly enough, though, Archimedes does not define the concept of center of gravity in any of his extant works. Archimedes probably developed the fundamentals in *On Centers of Gravity* and assumed these principles in successive writings.[8]

[8] [Heath] p. xxxvii

Archimedes illustrated his confidence in his work on levers with his well-known claim: δός μοι ποῦ στῶ καὶ τὰν γᾶν κινήσω ("Give me a place to stand, and I will move the earth"). King Hieron of Syracuse actually decided to test this. When the king had finished construction of an enormous ship, he called on Archimedes to launch the ship. While it took hundreds of men to originally move the ship, Archimedes moved it himself by means of a series of simple machines. This feat directly contradicted Aristotle's claim that a small force could not be distributed over a long time in such a way, as explained in the section above on Aristotle.

Due in part to his work with the ship, Archimedes was one of the most prominent figures in the field of mechanics. Both Plutarch and Pappus, a fourth century AD mathematician, credited Archimedes with discovering how to determine the mechanical advantage of levers, which resulted in catapults with much greater accuracy. This also gave birth to other contraptions that allowed force to be distributed and applied wherever most convenient, as in the example of the system Archimedes used to move Hieron's ship.[9]

One of the most famous stories about Archimedes is the following.[10] King Hieron of Syracuse was curious about how pure the gold was in his new crown and wanted to know whether the craftsman cheated him. Archimedes discovered a solution when he was taking a bath and realized that water displacement provided an accurate measure of volume. He was so excited that he ran out naked, shouting "Eureka," which means "I have discovered it." In this way Archimedes discovered buoyancy, the crux of one of his most influential works, *On Floating Bodies*. Demonstrating a direct application of this

9 [Russo] 71
10 Vitruvius, *Architect.* IX. 3

treatise, Archimedes invented the hydrostatic balance, which served as a way to find the specific gravity of a substance.

When his hometown of Syracuse was under attack by the Roman general Marcellus, Archimedes was called on to create new defenses for the city. Archimedes was the master designer behind accurate catapults and other contrivances that terrified the onslaught of Roman soldiers. Due to Syracuse's surprising resistance, Marcellus ordered his troops not to kill Archimedes and to lay a siege. Although Marcellus wanted to capture Archimedes alive, all accounts of his death involve a Roman soldier killing him out of rage. In one account, after Syracuse was conquered by Marcellus, a Roman soldier found Archimedes drawing in the sand. He commanded Archimedes to follow him; Archimedes refused, intent on continuing with his scratch work on the sand.[11] The frustrated Roman soldier killed him on the spot. Such an account exemplifies Archimedes's image as a focused theoretician.

Method

In 1906 the philologist Johan Ludvig Heiberg grew curious after reading Greek text that referred to a work of Archimedes in the same manuscript. Heiberg traveled to Constantinople in order to investigate the original manuscript. He soon discovered that the part of the parchment containing Archimedes's text had been scraped off in the 10th century in order to make room for other writing -- in other words, that the manuscript was a palimpsest. Fortunately, the scraping was not thorough enough to erase the words of Archimedes.[12]

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[11] [Dijksterhuis] p. 30
[12] [Heath] p. 5 of *Method* supplement

These newly discovered writings contained, among other things, Archimedes's treatise on *Method*, which had been thought lost. The work finally gives a glimpse into a prominent Greek theoretician's inductive process, which had never before been documented. Archimedes describes a mechanical heuristic whereby he treats components of geometric figures as levers, manipulating these as tools for discovering new areas to research. The text describes some simple theorems that use this principle, later building up to more complicated ones. Archimedes ends by giving the rigorous demonstration (*apodeixis*) of these ideas.

Archimedes makes it clear that even though his technique does not constitute a proof of theorems, it is a valuable tool for discovering new ideas that could later be proven rigorously. This method was a new inductive process that was neither widespread nor even accepted as proof. It is not clear why other ancient authors did so, but it was customary to leave no traces of the method that initially spawned the work to complete a proof.[13] *Method* thus allows us to see another facet of reasoning introduced in the Hellenistic Era.

Selected Text of Archimedes's Method

From *Method* I have extracted sections that illustrate Archimedes's purpose and the significance of his method. Below is the original text followed by my translation.

Ἀρχιμήδης Ἐρατοσθένει εὖ πράττειν.

Ἀπέστειλά σοι πρότερον τῶν εὑρημένων θεωρημάτων [06]
ἀναγράψας αὐτῶν τὰς προτάσεις φάμενος εὑρίσκειν
ταύτας τὰς ἀποδείξεις, ἃς οὐκ εἶπον ἐπὶ τοῦ παρόντος· [08]
ἦσαν δὲ τῶν ἀπεσταλμένων θεωρημάτων αἱ προτάσεις
αἵδε·
... Συμβαίνει δὲ [07]
ταῦτα τὰ θεωρήματα διαφέρειν τῶν πρότερον εὑρημένων·

[13] [Heath] p. 6-7 of *Method* supplement

ἐκεῖνα μὲν γὰρ τὰ σχήματα, τά τε κωνοειδῆ καὶ σφαιροειδῆ [09]
καὶ τὰ τμήματα <αὐτῶν, τῷ μεγέθει σχήμασι> κώνων
καὶ κυλίνδρων συνεκρίναμεν, ἐπιπέδοις δὲ περιεχομένῳ [11]
στερεῷ σχήματι οὐδὲν αὐτῶν ἴσον ἐὸν εὕρηται, τούτων
δὲ τῶν σχημάτων τῶν δυσὶν ἐπιπέδοις καὶ ἐπιφανείαις [13]
κυλίνδρων ἕκαστον ἑνὶ τῶν ἐπιπέδοις περιεχομένων
στερεῶν σχημάτων ἴσον εὑρίσκεται. [15]
Τούτων δὴ τῶν θεωρημάτων τὰς ἀποδείξεις ἐν τῷδε
τῷ βιβλίῳ γράψας ἀποστελῶ σοι. [17]
Ὁρῶν δέ σε, καθάπερ λέγω, σπουδαῖον καὶ φιλοσοφίας
προεστῶτα ἀξιολόγως καὶ τὴν ἐν τοῖς μαθήμασιν κατὰ [19]
τὸ ὑποπίπτον θεωρίαν τετιμηκότα ἐδοκίμασα γράψαι
σοι καὶ εἰς τὸ αὐτὸ βιβλίον ἐξορίσαι τρόπου τινὸς ἰδιότητα, [21]
καθ' ὅν σοι παρεχόμενον ἔσται λαμβάνειν ἀφορμὰς εἰς τὸ
δύνασθαί τινα τῶν ἐν τοῖς μαθήμασι θεωρεῖν διὰ τῶν [23]
μηχανικῶν. Τοῦτο δὲ πέπεισμαι χρήσιμον εἶναι οὐδὲν
ἧσσον καὶ εἰς τὴν ἀπόδειξιν αὐτῶν τῶν θεωρημάτων. [25]
Καὶ γάρ τινα τῶν πρότερόν μοι φανέντων μηχανικῶς
ὕστερον γεωμετρικῶς ἀπεδείχθη διὰ τὸ χωρὶς ἀποδείξεως [27]
εἶναι τὴν διὰ τούτου τοῦ τρόπου θεωρίαν· ἑτοιμότερον
γάρ ἐστι προλαβόντα διὰ τοῦ τρόπου γνῶσίν τινα τῶν [01]
ζητημάτων πορίσασθαι τὴν ἀπόδειξιν μᾶλλον ἢ μηδενὸς
ἐγνωσμένου ζητεῖν. <... Διόπερ καὶ τῶν θεωρη>μάτων [03]
τούτων, ὧν Εὔδοξος ἐξηύρηκεν πρῶτος τὴν ἀπόδειξιν,
περὶ τοῦ κώνου καὶ τῆς πυραμίδος, ὅτι τρίτον μέρος [05]
ὁ μὲν κῶνος τοῦ κυλίνδρου, ἡ δὲ πυραμὶς τοῦ πρίσματος,
τῶν βάσιν ἐχόντων τὴν αὐτὴν καὶ ὕψος ἴσον, οὐ μικρὰν [07]
ἀπονείμαι ἄν τις Δημοκρίτῳ μερίδα πρώτῳ τὴν ἀπόφασιν
τὴν περὶ τοῦ εἰρημένου σχήματος χωρὶς ἀποδείξεως [09]
ἀποφηναμένῳ. Ἡμῖν δὲ συμβαίνει καὶ τοῦ νῦν ἐκδιδομένου
θεωρήματος τὴν εὕρεσιν ὁμοίαν ταῖς πρότερον γεγενῆσθαι· [11]
ἠβουλήθην δὲ τὸν τρόπον ἀναγράψας ἐξενεγκεῖν ἅμα
μὲν καὶ διὰ τὸ προειρηκέναι ὑπὲρ αὐτοῦ, μή τισιν δοκῶμεν [13]
κενὴν φωνὴν καταβεβλῆσθαι, ἅμα δὲ καὶ πεπεισμένος
εἰς τὸ μάθημα οὐ μικρὰν ἂν συμβαλέσθαι χρείαν· ὑπο- [15]
λαμβάνω γάρ τινας ἢ τῶν ὄντων ἢ ἐπιγινομένων διὰ
τοῦ ἀποδειχθέντος τρόπου καὶ ἄλλα θεωρήματα οὔπω [17]
ἡμῖν συνπαραπεπτωκότα εὑρήσειν.
Γράφομεν οὖν πρῶτον τὸ καὶ πρῶτον φανὲν διὰ τῶν [19]
μηχανικῶν, ὅτι πᾶν τμῆμα ὀρθογωνίου κώνου τομῆς
ἐπίτριτόν ἐστιν τριγώνου τοῦ βάσιν ἔχοντος τὴν αὐτὴν [21]
καὶ ὕψος ἴσον, μετὰ δὲ τοῦτο ἕκαστον τῶν διὰ τοῦ αὐτοῦ
τρόπου θεωρηθέντων· ἐπὶ τέλει δὲ τοῦ βιβλίου γράφομεν [23]
τὰς γεωμετρι<κὰς ἀποδείξεις ἐκείνων τῶν θεωρημάτων,
ὧν τὰς προ>τάσεις ἀπεστείλαμέν <σοι πρότερον>. [25]

Translation

Archimedes wishes Eratosthenes to do well. I sent to you earlier (some) of the theorems discovered (by me), I having described the propositions of them and saying that I was discovering These proofs, which I had not spoken of during the present time; Of these theorems that I had sent, the propositions were the following;…

[Lines 10 through the end of page 82 and lines 1 through 6 on page 83 are skipped since their technical depth does not contribute to this deliverable.]

It happens that these theorems differ from the ones discovered previously; for those figures, both the conoids and the spheroids and the sections <of these,> we compared them <with respect to size with the figures> of cones and cylinders, but with respect to their planes, not one of them was found to be equal to a solid body surrounded by planes, but of these figures, of the cylinders with two planes and surfaces, each (of them) was found (to be) equal to one of the solid figures surrounded by planes. The proofs of these theorems in this book I am writing, I will send to you. Seeing that you, just as I say, stand out seriously and in a way that is worthy of philosophy and that you have honored the theory in mathematics as the occasion arises, I have decided to write to you and for the same book to define completely the idiosyncrasy of a certain method, according to which it will be provided for you to take the starting points in order to be able to θεωρεῖν (theorize) some of the (elements) in mathematics through mechanics. But I am persuaded that this is no less useful also for the proof of the theorems themselves. Furthermore, some of the things that first appeared to me by means of mechanics later were proved by means of geometry because the theory through this method is apart from the proofs; grasping some of the open questions through knowledge of the method, it is easier to provide the proof than to search knowing nothing. <Therefore of these theorems>, of which Eudoxus was the first to discover the proof, concerning the cone and the pyramid, that the cone is a third part of the cylinder, or the pyramid (is a third part of the) prism, both when they have the same base and equal height, someone might give Democritus no small part (credit), who first revealed a revelation concerning the figure discovered apart from proof. But it happens for us also that the discovery of the theorem now being published took place similarly to the previous (discoveries); but describing the method I wished on the one hand to publish on account of having spoken about it ahead of time, so that we do not seem to some to have made (thrown down) an empty voice (claim), but at the same time I was persuaded that it would contribute no small advantage (use) to learning; for I suppose that some of those being present or some of those coming (successors) through the proven method will also discover other theorems that have not yet occurred to us. We first write that which has been revealed has been revealed first through the mechanics, that every segment of a section of a right-angled cone is four thirds of a triangle having the same base and equal height, and after this each of the things through the same method have been observed; at the end of the book we write <the proofs of those theorems> with respect to geometry, (theorems) <whose proposition> we have sent <to you previously>.

Analysis of Text

Archimedes attempted to start a new inductive practice with his “method.” He chose to write to Eratosthenes because he trusted that Eratosthenes would make progress in the discipline: “you have honored the *theoria* (θεωρίαν) in mathematics as the occasion arises.”[14] Archimedes also had a working relationship with Eratosthenes, so he knew that Eratosthenes would listen to his idea. Furthermore, Archimedes believed that his method would be very useful as a starting point (ἀφορμὰς)[15] for further investigation, or, the starting point of a discovery process.

Archimedes used the word *theoria* several times in the text, drawing on its relevant etymological significance. For the Greeks, the word *theorēma* originally meant a religious vision that has been considered and held firm. It evolved from that form to its specific meaning “theorem” in mathematics. The word is derived from *theōros*, a pilgrim who was sent to consult an oracle. Later, the word designated someone who would behold an event, as in a theater. Finally, this word was associated with contemplating and considering; once this was used in a mathematical context, it referred to a theory or speculation. Archimedes used the term throughout because it could refer not only to those

[14] *Method* (page 83 lines 19-20)
[15] *Method* (page 83 line 22)

observations proven with an *apodeixis*, but also to those that have been speculated on due to his mechanical method.

Most importantly, Archimedes described his method as an *idiotās* (peculiarity). This word implies that it is Archimedes's own way of approaching mathematics, an idiosyncrasy. By using this language, he suggested that other people had not approached problems in a similar way. By publishing this as a text, Archimedes gave credit to the inductive process of discovery in addition to the deductive proof. Perhaps by equating his method as *idiotās*, he could get away with less criticism of the piece since he reported it as only his own.

Archimedes explained that it was a mechanical (μηχανικῶν)[16] method, both distinguishing it from the geometrical (γεωμετρικῶς)[17] method that he would later use to prove theorems and implying that the Greeks did not consider the mechanical method a sufficient *apodeixis* of the theorems proposed. Archimedes claimed that this method was useful for theorizing (θεωρεῖν)[18] even though the results were not technically *theoria*, which require *apodeixis*.

Archimedes used a rhetorical understatement construction three times in his opening statement. The first time, he implied that the mechanical method is "no less useful for the proof of the theorems themselves."[19] Later, he explained that Democritus should receive "no small credit"[20] for coming up with the revelation (ἀπόφασιν) of a theory but not the *apodeixis* for it. Finally, Archimedes claimed that the mechanical

[16] *Method* (page 83 line 24)
[17] *Method* (page 83 line 27)
[18] *Method* (page 83 line 23)
[19] *Metod* (page 83 lines 24-25)
[20] *Method* (page 84 line 7

method would result in "no small advantage to learning."[21] In using the rhetorical understatement three times, Archimedes clearly felt his method would have an impact on mathematical research.

It is important to note the significance of his method being mechanical—Archimedes found his inspiration by inserting levers into geometric figures and observing what to do in order to "balance" the figures. This proves that Archimedes did not confine his mathematical studies to the mathematical world. Rather, he used his practical ability to contribute to the development of the theory itself—a reversal of the established influence of theory on practice.

Conclusion

Archimedes discovered a new way of approaching geometry and mechanics, a methodology which built on existing reasoning and paved the way for progress. He was an accomplished mathematician, theoretician, and practicioner *because* he jumped the barriers of disciplines. His inductive process in the field of mechanics fueled growth and applications of the subject. His work *Method* gave other mathematicians new tools that developed not only geometrical studies, but also mechanical studies. By presenting his inductive process, he gave merit to the practice of jumping across the worlds of theory and application.

Interestingly enough, Archimedes's way of approaching research is not unlike our framework today. New technologies often drive research just as research can spawn a new field of technology—as an example, the microelectronics industry rapidly grew from such interplay in the 20th century. Today we also emphasize the interplay specifically between mathematics and its applications, with applications spawning new fields of math

[21] *Method* (page 84 line 15)

research. We are throwing our own modern “levers" into mathematics. Perhaps the methods of ancient theoreticians are more similar to our own methods today than is commonly thought.

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