ENGR 3399 Mechanical and Aerospace Systems I
Fall 2010
Assignment 1
Due Friday 9/10 at 5pm

You may work together on assignments but you must write up your own work.

1) Consider the *free* vibration of the single degree-of-freedom, mass-spring-damper system where x(t) describes the displacement of mass, m. The constants c and k are the linear damping and linear spring stiffness coefficients, respectively.

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = 0 \text{ with initial conditions } x(0) = x_0 \text{ and } \dot{x}(0) = v_0$$

First consider the *undamped* case, $c = 0$.
a) Write out the analytical solution for x(t)
b) What is the natural frequency?
c) Using m=1, k = 4, x_0 = 1, and v_0 =0, plot the free response, x(t) using MATLAB
d) Describe how the plot in c) would change if k is increased and decreased. How about if m is increased and decreased?

Now consider the *damped* case, $c \neq 0$
e) Write out the analytical solution for x(t).
f) Using m=1, c = 0.1, k = 4, x_0 = 1, and v_0 =0, plot the free response, x(t).
g) Describe how the plot in part f) would change if c is increased and decreased.

2) Using the MATLAB function, ODE45, directly integrate the single d-o-f , harmonic oscillator differential equation for all of the cases in question 3. Make both displacement and velocity time-series plots. Compare these to those generated from the analytical solutions. Discuss differences.

3) Using SIMULINK, create a model of the single d-o-f harmonic oscillator. Through the oscilloscope icon, make plots of the displacement and velocity response. Verify that these results are the same as those generated by direct numerical integration. (Obviously, they should be the same as MATLAB is doing the same calculations). Hand in copy of the system diagram.

4) Consider the *forced* vibration of the single d-o-f mass-spring-damper system.

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t) \text{ with } x(0) = x_0 \text{ and } \dot{x}(0) = v_0$$

For the applied (to the mass) excitation, $f(t) = A\sin(\omega_f t)$,
a) Write out the analytical solution for x(t).
b) Using m=1, c=0, k=4, f(t) = sin3t, x_0 = 1, and v_0 =0, plot the total response, x(t).

c) Using m=1, c=0, k=4, f(t) = sin2t, x_0 = 1, and v_0 =0, plot the total response, x(t).
Important note: what happens in this case to the solution found in part a? What can be done to find an analytical solution?
d) Using m=1, c=0.1, k=4, f(t) = sin2t, x_0 = 1, and v_0 =0, plot the total response, x(t).