MTH 2140 Syllabus

Differential Equations, Fall 2010, Session I

Mθ 1:00 - 2:50 pm in AC 126

Instructor: Aaron Hoffman
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Office hours: W 1-3 in MH 257[1] and θ 12-1 in the dining hall
Textbook: Differential Equations, Blanchard, Devaney, and Hall, 3rd ed.

About this Class:

MTH 2140 is a first course in differential equations. We will study how to use differential equations to model physical systems, how to find exact solutions of differential equations in a few special cases, and how to make precise statements about the qualitative behavior of solutions to differential equations. The main themes of the course follow:

- For most differential equations, we cannot write down any solution in terms of a formula. This doesn't mean that the solutions don't exist or aren't functions. It just means that the function is defined in terms of a differential equation. Moreover, it doesn't mean we can't study these functions. In fact ...
- Rough qualitative information about solutions of a differential equation can often be obtained by looking at very coarse information concerning the differential equation, such as the sign of the function(s) on the right hand side.
- A consequence of the above is that the rough qualitative behavior of solutions of differential equations is typically robust with respect to small changes in a parameter. However, there are situations in which a small change in the parameter value can result in large changes in the qualitative behavior of solutions (think $\dot{x} = ax$ as a goes from positive to negative). These parameter values are called bifurcation points. Studying them lets us partition the parameter space into regions defined by the qualitative behavior of solutions.
- There are certain differential equations whose solutions we can write down explicitly (e.g. the solution to $\dot{x} = ax$ is Ce^{at} for some constant C which does not depend on time). The equations that we will study in this class for which exact solutions exist are the linear constant coefficient equations. Solutions of such equations are always built from exponentials because exponentials play well with the combination of differentiation and multiplication by a constant.
- Linear differential equations are based on linear algebra. I will teach you the linear algebra that I expect you to know.
- All differential equations can be thought of as linear + bells and whistles. Understanding the linear theory allows us to say a lot about nonlinear equations. The example that we will discuss in this class is the technique of linearization whereby the near-equilibrium dynamics of a nonlinear system are well-approximated by the dynamics of the linear system which best approximates the nonlinear system near the equilibrium.

[1] Nov 3 and Dec 1 office hours will be cancelled

Feedback:
This document describes a tentative syllabus and course plan. It is likely that we will deviate from this plan in part due to feedback that I'm getting from you. To that end, please give me feedback. Tell me what is working for you and what is not working for you. Give me suggestions for new things to try. I cannot promise to act on all of your suggestions, but I do promise to listen and incorporate your feedback into my teaching.

Grading and Assessment:
Your grade will be based on weekly homework assignments (30 %), weekly quizzes (30 %) and a final (35 %) as well as a small subjective component (5 %) which takes into account class participation, effort and attitude.

Homework: Homework will be assigned weekly on Friday and due the following Friday.

- Classmates are encouraged to work together on the homework. However, each individual must hand in his or her own homework assignment and see the next bullet point on citation.
- Cite your sources. A source is any classmate, teaching assistant, friend, or instructor who helps you with a problem. A citation is a brief note which explains who helped you and how they helped you. This is a useful habit for the sake of academic honesty. It is also helpful because you will have a record of which problems you were able to solve by yourself and which problems required help which might help you direct your energies when it comes time to review the material. Collect the citations together in a brief section at the end of your homework.
- Work should be neat and legible and writing should be clear. You should not have the remnants of spiral notebook pages hanging from your assignment. Unless otherwise specified, your audience will be a mathematical expert. You will be expected to write clearly enough so that an expert can understand what you've done and easily grade your work. For some problems, I will ask that you write for an audience consisting of mathematical peers. In this case you will be assessed based on the clarity of your exposition.

Quizzes:
Quizzes will be given weekly. They will be take-home and closed book and will consist of one or two problems which will be similar in spirit to a homework problem of medium-level difficulty. The purpose of the quizzes is (i) to encourage you to learn the material before the night before the final (ii) to give consistent feedback. Quizzes will be cumulative up to the material on the previous week's homework. After you have completed the quiz, you can rework it in an open-book (but still closed-human resource) environment in a different pen color. The open-book work will receive half credit.

Final:
The final exam will be take-home and closed book. It will be made available after our last class on Thursday December 9 and will be due 5:00 PM on Tuesday December 14. It will consist of a range of problems, some of which will be drill-like and others of which will require creativity and problem-solving techniques. You will have three hours to complete the exam. After you have done as well as you can in a closed-book environment or three hours have expired (whichever comes

first), you can change pen color and work for an additional hour in an open-book open-notes environment for half credit.

Partial Credit:

If you cannot complete a problem, do not fudge it and hope for partial credit. I don't expect that you'll get every problem, but I will be able to give you better feedback if I know what stumbling blocks you've hit. Write a sentence or two about why this particular problem was difficult for you to complete. Is there a computation that you don't know how to carry out? Is there an abstract concept that you are having a tough time wrapping your mind around? Is there a line of reasoning that you can't work out? Did you overrun the amount of time that you had budgeted for this problem? Be as specific as possible. Such an explanation by itself does not guarantee partial credit; however it is a necessary condition: partial credit for incomplete work will only be awarded if it is accompanied by such an explanation.

Tentative Outline:

- Week 1 [Oct 21]
  - Modeling; important models. [Section 1.1]
  - Introduction to techniques of solution: separation of variables and slope fields [Sections 1.2-1.3]
- Week 2 [Oct 25, Oct 28]
  - existence and uniqueness [Section 1.5]
  - geometric and topological consequences of existence and uniqueness. [Sections 1.6-1.7]
- Week 3 [Nov 1, Nov 4]
  - Nonautonomous linear scalar equations [Sections 1.8-1.9]
  - Second order linear equations [Damped and driven harmonic oscillators (includes material from chapter 4)]
  - nonautonomous linear equations [Bessel's equation; power series solutions]
- Week 4 [Nov 8, Nov 11, Nov 12]
  - The phase plane [Chapter 2]
  - Linear systems [Chapter 3]
- Week 5 [Nov 15, Nov 18]
  - Finish up linear systems [Chapter 3]
  - Change of basis
  - Matrix exponential
  - Vector Duhamel formula
- Week 6 [Nov 29, Dec 2] Linearization; Nullclines [Sections 5.1 and 5.2]
- Week 7 [Dec 6, Dec 9] Hamiltonian systems, Lyapunov functions, and gradient systems [Sections 5.3 and 5.4]
- With any extra time we will talk about N-dimensional systems, ∞-dimensional systems, and partial differential equations.