Shape stability of sonoluminescence bubbles: Comparison of theory to experiments

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Single bubble sonoluminescence (SBSL) is the brief flash of light emitted from a single, stable, acoustically forced bubble. In experiments, the maximum pressure amplitude with which a bubble may be forced is limited by considerations of spherical stability. The traditional linear stability analysis predicts a threshold for SBSL at a much lower pressure amplitude than experimental observations. This work shows that if one constructs an accurate model of the radial dynamics, the traditional linear stability analysis predicts a boundary that is in excellent agreement with experimental data.

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I. INTRODUCTION

A single gas bubble can be levitated in a liquid container and forced into violent radial oscillations by an acoustic field: typical of the nonlinear bubble dynamics is a slow expansion followed by a violent collapse. At high acoustic driving the collapse can become violent enough that the rapidly compressed gas is hot enough to emit a brief flash of light. This unusual phenomena is known as single bubble sonoluminescence (SBSL). The experimental parameter space where this phenomena is observable is limited by considerations of dissolved gas diffusion, chemical reactions, and spherical shape stability: mapping this parameter space was covered in detail by Hilgenfeldt *et al.* [1]. This paper is concerned with the theory used to predict the shape stability threshold.

Plesset [2] analyzed the spherical symmetry of a bubble undergoing volume oscillations and derived the equations to predict the evolution of non-spherical disturbances. This analysis was furthered over the years (see, e.g., Refs. [3,4]) and recently this linear theory was applied to SBSL experiments [4–6].

There are two shape stability thresholds in SBSL: parametric stability and Rayleigh-Taylor stability. Parametric instability is a long-time scale phenomena where the nonspherical perturbations accumulate and grow from one acoustic cycle to the next. Rayleigh-Taylor instability occurs in very violent collapses where the nonspherical perturbations do not grow from cycle to cycle but simply become too large in one collapse [1]. The current work is only concerned with parametric stability, as it is the commonly measured threshold in experiments.

Previous work found that the classical linear stability theory underpredicts the experimentally measured stability boundary in SBSL. The linear stability theory predicts that the $n=2$ spherical harmonic shape mode is the most unstable, while data on SBSL are found to agree more closely with the n=4 mode. This result caused speculation that nonlinear (or other unknown) effects may be important in SBSL stability [5]. Hao and Prosperetti [4], however, showed that the predicted location of the spherical stability boundary was very sensitive to details of the radial dynamics: strongly damped oscillations were found to have a larger stable region.

This paper will demonstrate that when one constructs an accurate model of the radial dynamics the linear stability analysis of the $n=2$ mode agrees with experimental data. A model from previous work which involves direct numerical simulation (DNS) of the gas dynamics of violent bubble collapses will be used to compute the radial dynamics and subsequently determine the stability boundary [7]. The stability boundary calculated with the DNS agrees with the experimental data supplied by Ketterling [8]. The conclusion of the current paper is that the inability of traditional analysis to accurately predict the stability boundary is related to the inability of traditional models of radial dynamics to accurately capture the damping of the oscillations. This result confirms the result of Hao and Prosperetti [4], who also found that the stability boundary is very sensitive to the radial dynamics.

II. FORMULATION

If one takes the Navier-Stokes equations for the liquid surrounding a bubble and assumes that the liquid is incompressible (or only ''mildly'' compressible) and that the bubble remains spherically symmetric, it is straightforward to derive a nonlinear ordinary differential equation (ODE) for the bubble radius. To make this ODE solvable, one must know the pressure inside the bubble.

Traditionally, one assumes the pressure inside the bubble is uniform and that the bubble undergoes a polytropic process (i.e., isothermal or adiabatic). This assumption directly relates the gas pressure to the volume, and makes the ODE solvable: this equation is known as the Rayleigh-Plesset equation (RPE). A more accurate model uses a one-dimensional direct numerical simulation of the gas dynamics to compute the gas pressure. The DNS solves the complete Navier-Stokes equations of a multispecies gas including phase change, heat transfer, mass transfer, and chemical reactions. The equations used in the DNS were described in detail by Storey and Szeri [7], and therefore will not be repeated here. This DNS method was found in the previous work to produce radial oscillations that agree well with experiments. It is important to realize the two methods are modeling the liquid in the same manner: the difference is that the DNS solves the full equations governing the gas,

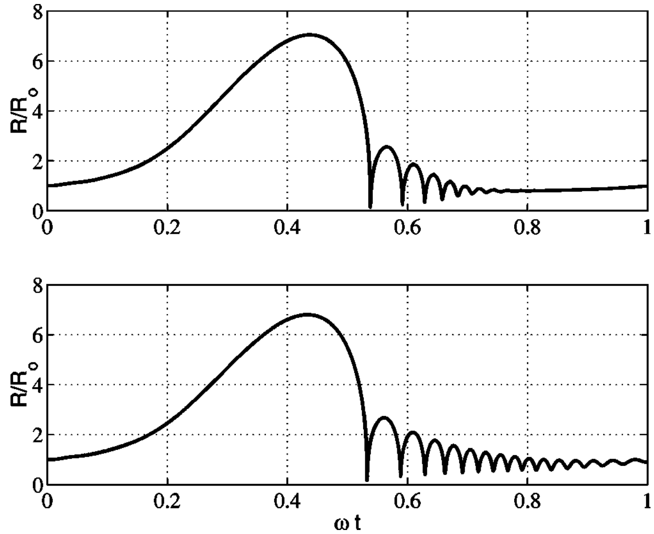

FIG. 1. Radial dynamics from a DNS model and a traditional RPE model. The conditions are a 4.5-micron bubble forced with a 1.3-atm. pressure amplitude at 32.8 kHz. The top figure is from DNS's, and the bottom figure is from the RPE. The radius is scaled by the ambient radius, and time is scaled by the frequency of the forcing.

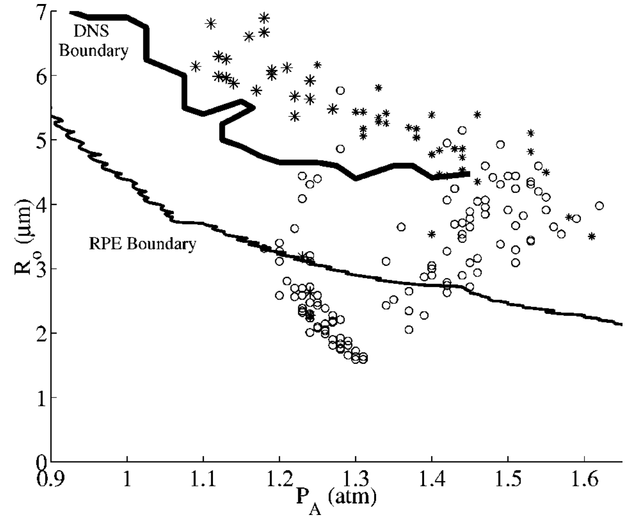

FIG. 2. Comparison of the stable parameter space (ambient radius R_0 and acoustic pressure amplitude P_A) of the DNS, RPE, and experimental data. The thin line is the stability boundary for the RPE, while the thick line is the boundary from DNS: bubbles above the curves are unstable. The experimental data points are a subset of that presented in Fig. 5 of Ketterling and Apfel. The experimental data are for air only, with the open circles representing stable points and the stars unstable points. Only air data are plotted, since this theory will predict slightly different boundaries depending on the nature of the gas. The linear stability theory using DNS shows excellent agreement with the experimental data.

whereas the RPE uses a uniform pressure and polytropic assumption.

The equations which govern the evolution of nonspherical perturbations on the bubble interface were derived by Plesset [2], and have since been modified to the form commonly used today [4]. The linear stability analysis predicts that the $n=2$ spherical harmonic mode is the most unstable. Further descriptions and derivations of these equations can be found in numerous references [4,1,6] but the equations will not be repeated here in the interest of brevity. Note that the equation for the non-spherical growth rate is a function of the radial dynamics $[R(t),\ \dot{R}(t)$, and $\ddot{R}(t)]$ and physical properties of the liquid.

III. RESULTS

In order to compare the DNS model to experiments, the recent data of Ketterling and Apfel [5] are modeled. The liquid temperature and ambient pressure are room conditions (1 atm and 293 K), and the frequency of the acoustic forcing is 32.8 kHz. The ambient radius and pressure amplitude are varied to map the parameter space of stable and unstable bubbles. Note that, in experiments, the ambient radius is controlled by varying the pressure amplitude and the dissolved gas concentration.

In Fig. 1, the radial oscillations for the two models (the DNS and traditional RPE models) are plotted in order to highlight the practical difference in the two approaches. Radial oscillations from the DNS model are found to damp much more rapidly when compared to the traditional RPE model with an isothermal assumption. The extra damping in the DNS case is not surprising, since the full simulation accounts for transport and dissipation of heat and mass in the bubble, a source of loss for the energy of the collapse. It was previously shown that such DNS's provide good agreement with experimental observations in terms of the radial dynamics [7,9]. There are other approaches that provided similar dampings of radial oscillations [10].

In Fig. 2 the stability boundaries from the DNS and RPE models, are compared to experimental data. Note that there are a variety of versions of the RPE that are derived with various assumptions and approximations. The form of the RPE used in this work is identical to the one used by Hilgenfeldt *et al.* [1], simply for comparison purposes and the choice is not important to the result. The thin line on the lower part of the graph is the stability boundary computed from the traditional RPE: bubbles above the curve are predicted to be unstable, and bubbles below to be stable. The heavy line is the stability boundary computed via the DNS. The experimental data are the plotted points where the open circles are stable and the stars are unstable points. One can clearly see that the DNS model agrees very well with the experiments, especially in relation to the RPE model.

The original data set [5] included a variety of dissolved gases, but only data for argon bubbles were plotted in Fig. 2. The identity of the gas influences the radial dynamics, and subsequently has a minor effect on the stability boundary. For example, helium has much longer-lived afterbounces than argon: this fact is observed experimentally and seen with the DNS model. The difference in the afterbounce behavior is a direct indication of heat and mass transfer in the gas influencing the radial dynamics of the bubble. This change in the radial dynamics causes the helium stability boundary to be lowered by 0.25–0.5 μm in a pressure amplitude range between 1.2 and 1.5 atm. This minor difference between argon and helium is smaller than current experimental uncertainty, and therefore cannot be resolved.

Due to the computational time it takes to calculate one acoustic cycle with the DNS model, this stability boundary is more coarsely resolved than the boundary computed with the RPE model. The theoretical stability boundaries can have many fine features, but these features are of no practical importance as they cannot be measured due to practical limits of experimental uncertainty. Also note that the DNS results only extend to about 1.5 atm of pressure amplitude. Above this limit the gas dynamics become so violent that computing an entire cycle with the current DNS method becomes intractable with the current resources.

The trend in the numerical result is clear, the more realistic damping in the DNS simulations moves the stability boundary to larger initial radii for a given pressure amplitude. The results from the DNS model are in excellent agreement with the experimental data. This agreement demonstrates that the linear stability theory is in fact accurate for these violent oscillations, and nonlinear theories are not needed to explain the parametric spherical stability limits.

IV. CONCLUSIONS

The parametric spherical stability of a SBSL bubble was investigated with detailed modeling of the gas dynamics in the bubble interior. The DNS model of radial dynamics was compared to traditional RPE simulations, and the DNS produced radial oscillations that agree well with experiments. The added damping of the radial oscillations in the DNS model stabilizes the spherical symmetry of the bubble. The parameter space diagram of stable and unstable bubbles computed with the gas dynamics DNS model coupled to the linear stability theory agrees well with the experiments of Ketterling [8].

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